Paper 3A: Further Pure Mathematics 1 Mark Scheme

Question	Scheme	Marks	AOs
1	Step 0.5	B1	1.1b
	<table><tr><td></td><td>y_0</td><td>y_1</td><td>y_2</td><td>y_3</td><td>y_4</td><td>y_5</td><td>y_6</td></tr><tr><td>x</td><td>1</td><td>1.5</td><td>2</td><td>2.5</td><td>3</td><td>3.5</td><td>4</td></tr><tr><td>y</td><td>$\sqrt{2}$</td><td>$\sqrt{4.375}$</td><td>3</td><td>$\sqrt{16.625}$</td><td>$\sqrt{28}$</td><td>$\sqrt{43.875}$</td><td>$\sqrt{65}$</td></tr></table>	M1	1.1b
	$y_0+4y_1+2y_2+4y_3+2y_4+4y_5+y_6="77.23"$	M1	1.1b
	$\int_1^4 \sqrt{1+x^3}\,dx \approx \frac{0.5}{3}\times"77.23"$	M1	1.1b
	$= 12.9$	A1	1.1b
		(5)	
			(5 marks)

Notes:

B1: Use of step length 0.5
M1: Attempt to find y values with at least 2 correct
M1: Use of formula "$y_0+4y_1+2y_2+4y_3+2y_4+4y_5+y_6$" with correct coefficients
A1: $\frac{0.5}{3}\times$ their 77.23
A1: awrt 12.9

Question	Scheme	Marks	AOs
2	$y = x^3 e^{kx}$ so $u = x^3$ and $\frac{du}{dx} = 3x^2$ and $\frac{d^2u}{dx^2} = 6x$ and $\frac{d^3u}{dx^3} = 6$ $\left(\text{and } \frac{d^4u}{dx^4} = 0\right)$	M1	1.1b
	$v = e^{kx}$ and $\frac{d^nv}{dx^n} = k^n e^{kx}$ and $\frac{d^{n-1}v}{dx^{n-1}} = k^{n-1}e^{kx}$ and $\frac{d^{n-2}v}{dx^{n-2}} = k^{n-2}e^{kx}$ (and...)	M1	2.1
	$\frac{d^ny}{dx^n} = x^3k^ne^{kx} + n3x^2k^{n-1}e^{kx} + \frac{n(n-1)}{2}6xk^{n-2}e^{kx} + \frac{n(n-1)(n-2)}{3!}6k^{n-3}e^{kx}$ and remaining terms disappear	M1	2.1
	So $\frac{d^ny}{dx^n} = k^{n-3}e^{kx}\left(k^3x^3 + 3nk^2x^2 + 3n(n-1)kx + n(n-1)(n-2)\right)$ *	A1*	1.1b
		(4)	
		(4 marks)	

Notes:

M1: Differentiate $u = x^3$ three times

M1: Use $u = e^{kx}$ and establish the form of the derivatives, with at least the three shown

M1: Uses correct formula, with 2 and 3! (or 6) and with terms shown to disappear after the fourth term

A1*: Correct solution leading to the given answer stated. No errors seen

Question	Scheme	Marks	AOs
3(a)	Use of $x = tv$ to give $\frac{\mathrm{d}x}{\mathrm{d}t} = v + t\frac{\mathrm{d}v}{\mathrm{d}t}$	M1	1.1b
	Hence $\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = \frac{\mathrm{d}v}{\mathrm{d}t} + \frac{\mathrm{d}v}{\mathrm{d}t} + t\frac{\mathrm{d}^2v}{\mathrm{d}t^2}$	M1	2.1
		A1	1.1b
	Uses $t^2\left(\text{their } 2^{\text{nd}}\text{derivative}\right) - 2t\left(\text{their } 1^{\text{st}}\text{derivative}\right) + (2+t^2)x = t^4$ and simplifies LHS	M1	2.1
	$\left(t^3\frac{\mathrm{d}^2v}{\mathrm{d}t^2} + t^3v = t^4 \text{ leading to}\right)$ $\frac{\mathrm{d}^2v}{\mathrm{d}t^2} + v = t$ *	A1*	1.1b
		(5)	
(b)	Solve $\lambda^2 + 1 = 0$ to give $\lambda^2 = -1$	M1	1.1b
	$v = A\cos t + B\sin t$	A1ft	1.1b
	Particular Integral is $v = kt + l$	B1	2.2a
	$\frac{\mathrm{d}v}{\mathrm{d}t} = k$ and $\frac{\mathrm{d}^2v}{\mathrm{d}t^2} = 0$ and solve $0 + kt + l = t$ to give $k = 1,\ l = 0$	M1	1.1b
	Solution: $v = A\cos t + B\sin t + t$	A1	1.1b
	Displacement of C from O is given by $x = tv = \ldots$	M1	3.4
	$x = t\left(A\cos t + B\sin t + t\right)$	A1	2.2a
		(7)	
(c)(i)	For large t, the displacement gets very large (and positive)	B1	3.2a
(ii)	Model suggests midpoint of spring moving relative to fixed point has large displacement when t is large, which is unrealistic. The spring may reach elastic limit / will break	B1	3.5a
		(2)	
			(14 marks)

Question 3 notes:	
(a)	
M1:	Uses product rule to obtain first derivative
M1:	Continues to differentiate again, with product rule and chain rule as appropriate, in order to establish the second derivative
A1:	Correct second derivative. Accept equivalent expressions
M1:	Shows clearly the substitution into the given equation in order to form the new equation and gathers like terms
A1*:	Fully correct solution leading to the given answer
(b)	Accept variations on symbols for constants throughout
M1:	Form and solve a quadratic Auxiliary Equation
A1ft:	Correct form of the Complementary Function for their solutions to the AE
B1:	Deduces the correct form for the Particular Integral (note $v = mt^2 + kt + l$ is fine)
M1:	Differentiates their Particular Integral and substitutes their derivatives into the equation to find the constants (m = 0 if used)
A1:	Correct general solution for equation (II)
M1:	Links the solution to equation (II) to the solution of the model equation correctly to find the displacement equation
A1:	Deduces the correct general solution for the displacement
(c)(i)	
B1:	States that for large t the displacement is large o.e. Accept e.g. as $t \to \infty$, $x \to \infty$
(c)(ii)	
B1:	Reflect on the context of the original problem. Accept ‘model unrealistic’ / ‘spring will break’

Question	Scheme	Marks	AOs
4(a)	$y''=2xy'-y \Rightarrow y'''=2xy''+2y'-y'$	M1	1.1b
		A1	1.1b
	$y'''=2xy''+y' \Rightarrow y''''=2xy'''+2y''+y''$	M1	2.1
	$y''''=2xy'''+3y'' \Rightarrow y'''''=2xy''''+5y'''$	A1	2.1
		(4)	
(b)	$x=0, y=0, y'=1 \Rightarrow y''(0)=0\pi$ from equation	B1	2.2a
	$y'''(0)=2\times0\times y''(0)+1=1$; $y''''(0)=2\times0\times1+3\times0=0$;	M1	1.1b
	$x=0,\ y'''(0)=1,\ y''''(0)=0 \Rightarrow y'''''(0)=5$	A1	1.1b
	$y=y(0)+y'(0)x+\dfrac{y''(0)}{2}x^2+\dfrac{y'''(0)}{6}x^3+\dfrac{y''''(0)}{24}x^4+\dfrac{y'''''(0)}{120}x^5+...$	M1	2.5
	Series solution: $y=x+\dfrac{1}{6}x^3+\dfrac{1}{24}x^5+...$	A1ft	1.1b
		(5)	
			(9 marks)

Notes:

(a)

M1: Attempts to differentiate equation with use of the product rule

A1: cao. Accept if terms all on one side

M1: Continues the process of differentiating to progress towards the goal. Terms may be kept on one side, but an expression in the fourth derivative should be obtained

A1: Completes the process to reach the fifth derivative and rearranges to the correct form to obtain the correct answer by correct solution only

(b)

B1: Deduces the correct value for $y''(0)$ from the information in the question

M1: Finds the values of the derivatives at the given point

A1: All correct

M1: Correct mathematical language required with given denominators. Can be in factorial form

A1ft: Correct series, must start $y = ….$ Follow through the values of their derivatives at 0

Question	Scheme	Marks	AOs
5	$y^2 = 4ax \Rightarrow 2y\frac{dy}{dx} = 4a$	M1	2.1
	$\frac{dy}{dx} = \frac{2a}{y} \Rightarrow$ Gradient of normal is $\frac{-y}{2a} = -p$	A1	1.1b
	Equation of normal is : $y - 2ap = -p(x - ap^2)$	M1	1.1b
	Normal passes through $Q(aq^2, 2aq)$ so $2aq + apq^2 = 2ap + ap^3$	M1	3.1a
	Grad OP $\times$ Grad $OQ = -1 \Rightarrow \frac{2ap}{ap^2}\frac{2aq}{aq^2} = -1$	M1	2.1
	$q = \frac{-4}{p}$	A1	1.1b
	$2a\left(\frac{-4}{p}\right) + ap\left(\frac{16}{p^2}\right) = 2ap + ap^3 \Rightarrow p^4 + 2p^2 - 8 = 0$	M1	2.1
	$(p^2 - 2)(p^2 + 4) = 0 \Rightarrow p^2 = ...$	M1	1.1b
	Hence (as $p^2 + 4 \neq 0$), $p^2 = 2$*	A1*	1.1b
		(9)	
	Alternative 1 First three marks as above and then as follows	M1 A1 M1	2.1 1.1b 1.1b
	Solves $y^2 = 4ax$ and their normal simultaneously to find, in terms of a and p, either $x_Q\left(= ap^2 + 4a + \frac{4a}{p^2}\right)$ or $y_Q\left(= -2ap - \frac{4a}{p}\right)$	M1	3.1a
	Finds the second coordinate of Q in terms of a and p	M1	1.1b
	Both $x_Q = ap^2 + 4a + \frac{4a}{p^2}$ and $y_Q = -2ap - \frac{4a}{p}$	A1	1.1b
	Grad OP $\times$ Grad $OQ = -1 \Rightarrow \frac{2ap}{ap^2} \times \frac{-2ap - \frac{4a}{p}}{ap^2 + 4a + \frac{4a}{p^2}} = -1$	M1	2.1
	Simplifies expression and solves: $4p^2 + 8 = p^4 + 4p^2 + 4$ $\Rightarrow p^4 - 4 = 0 \Rightarrow (p^2 - 2)(p^2 + 2) = 0 \Rightarrow p^2 = ...$	M1	2.1
	Hence (as $p^2 + 2 \neq 0$), $p^2 = 2$*	A1*	1.1b
		(9)	

Question	Scheme	Marks	AOs
5	**Alternative 2** First three marks as above and then as follows	M1 A1 M1	2.1 1.1b 1.1b
	Solves $y^2 = 4ax$ and their normal simultaneously to find, in terms of a and p, either $x_Q\left(= ap^2 + 4a + \frac{4a}{p^2}\right)$ or $y_Q\left(= -2ap - \frac{4a}{p}\right)$	M1	3.1a
	Forms a relationship between p and q from their first coordinate: **either** $y_Q = 2a\left(-p - \frac{2}{p}\right) \Rightarrow q = -p - \frac{2}{p}$ **or** $x_Q = a\left(p + \frac{2}{p}\right)^2 \Rightarrow q = \pm\left(p + \frac{2}{p}\right)$	M1	2.1
	$q = -p - \frac{2}{p}$ (if x coordinate used the correct root must be clearly identified before this mark is awarded)	A1	1.1b
	Grad OP × Grad $OQ = -1 \Rightarrow \frac{2ap}{ap^2} \times \frac{2aq}{aq^2} = -1 \left(\Rightarrow q = -\frac{4}{p}\right)$	M1	2.1
	Sets $q = -p - \frac{2}{p} = -\frac{4}{p}$ and solves to give $p^2 = ...$	M1	1.1b
	Hence $\left(\text{as } q = p + \frac{2}{p} = -\frac{4}{p} \text{ gives no solution}\right)$, $p^2 = 2$ (only)*	A1*	1.1b
		(9)	
		(9 marks)	

Notes:

(a)

M1: Begins proof by differentiating and using the perpendicularity condition at point P in order to find the equation of the normal

A1: Correct gradient of normal, $-p$ only

M1: Use of $y - y_1 = m(x - x_1)$. Accept use of $y = mx + c$ and then substitute to find c

M1: Substitute coordinates of Q into their equation to find an equation relating p and q

M1: Use of $m_1 m_2 = -1$ with OP and OQ to form a second equation relating p and q

A1: $q = \frac{-4}{p}$ only

M1: Solves the simultaneous equations and cancels a from their results to obtain a quadratic equation in p^2 only

M1: Attempts to solve their quadratic in p^2. Usual rules

A1*: Correct solution leading to given answer stated. No errors seen

Question 5 notes continued:

Alternative 1:

M1A1M1: As main scheme

M1: Solves $y^2 = 4ax$ and their normal simultaneously to find one of the coordinates for Q in terms of a and p as shown

M1: Finds the second coordinate of Q in terms of a and p

A1: Both coordinates correct in terms of a and p

M1: Use of $m_1 m_2 = -1$ with OP and OQ. i.e. $\frac{2ap}{ap^2} \times \frac{\text{their } y_Q}{\text{their } x_Q} = -1$ with coordinates of P and their expressions for x_Q and y_Q

M1: Cancels the a's, simplifies to a quadratic in p^2 and solves the quadratic. Usual rules

A1*: Correct solution leading to the given answer stated. No errors seen

Alternative 2:

M1A1M1: As main scheme

M1: Solves $y^2 = 4ax$ and their normal simultaneously to find one of the coordinates for Q in terms of a and p as shown

M1: Uses their coordinate to form a relationship between p and q. Allow $q = \left(p + \frac{2}{p}\right)$ for this mark

A1: For $q = -p - \frac{2}{p}$. If the x coordinate was used to find q then consideration of the negative root is needed for this mark. Allow for $q = \pm\left(p + \frac{2}{p}\right)$

M1: Use of $m_1 m_2 = -1$ with OP and OQ to form a second equation relating p and q only

M1: Equates expressions for q and attempts to solve to give $p^2 = ...$

A1*: Correct solution leading to the given answer stated. No errors seen. If x coordinate used, invalid solution must be rejected

Question	Scheme	Marks	AOs
6(a)	$\mathbf{AB}\times\mathbf{AC}=\begin{pmatrix}-1\\-1\\-1\end{pmatrix}\times\begin{pmatrix}1\\-1\\2\end{pmatrix}=\begin{pmatrix}-2-1\\-1+2\\1+1\end{pmatrix}=\begin{pmatrix}-3\\1\\2\end{pmatrix}$	M1	1.1b
	$\mathbf{r}.\begin{pmatrix}-3\\1\\2\end{pmatrix}=\begin{pmatrix}0\\1\\0\end{pmatrix}.\begin{pmatrix}-3\\1\\2\end{pmatrix}=1$	M1	1.1b
	Hence $-3x+y+2z=1$	A1	1.1b
		(3)	
(b)	Volume of Tetrahedron = $\frac{1}{6}\lvert\mathbf{n}.(\mathbf{AD})\rvert$	M1	3.1a
	$=\frac{1}{6}\left\lvert\begin{pmatrix}-3\\1\\2\end{pmatrix}.\left(\begin{pmatrix}10\\5\\5\end{pmatrix}-\begin{pmatrix}1\\2\\1\end{pmatrix}\right)\right\rvert$	M1	1.1b
	$=\frac{1}{6}\lvert(-27+3+8)\rvert=\frac{8}{3}$	A1	1.1b
		(3)	
(c)	$\mathbf{AE}=k\mathbf{AC}$ so E is $(1+k,\ 2-k,\ 1+2k)$	M1	3.1a
	E lies on plane so $2(1+k)-3(2-k)+3=0$, leading to $k=...$	M1	3.1a
	Hence $k=\frac{1}{5}$	A1	1.1b
		(3)	
(d)	Volume $ABEF$ = $\frac{1}{6}(\mathbf{AB}\times\mathbf{AE}).\mathbf{AF}=\frac{1}{6}\left(\mathbf{AB}\times\frac{1}{5}\mathbf{AC}\right).\frac{1}{9}\mathbf{AD}$	M1	3.1a
	$=\frac{1}{45}\left(\frac{1}{6}(\mathbf{AB}\times\mathbf{AC}).\mathbf{AD}\right)$ and hence result *	A1*	2.2a
		(2)	
		(11 marks)	

Question 6 notes:	
(a)	
M1:	Attempting a suitable cross product. Accept use of unit vectors
M1:	Complete method that would lead to finding the Cartesian equation of plane
A1:	Accept any equivalent form
(b)	
M1:	Identifies suitable vectors and attempts to substitute into a correct formula. Accept use of unit vectors
M1:	Correct form of scalar triple product using their **n** from part (a)
A1:	$\frac{8}{3}$ or exact equivalent form
(c)	
M1:	Uses that *E* is on *AC* in order to find an expression for *E*
M1:	Uses that *E* is in the plane Π to form and solve an expression in *k*
A1:	$\frac{1}{5}$ o.e. only
(d)	
M1:	Uses formula for volume of tetrahedron and substitutes for $\mathbf{AE}$ and $\mathbf{AF}$
A1*:	Deduces result: Use of $\frac{1}{6}(\mathbf{AB} \times \mathbf{AC}).\mathbf{AD}$ is required and no errors seen in solution

Question	Scheme	Marks	AOs
7	$x^2+4y^2=4 \Rightarrow 2x+8y\frac{\mathrm{d}y}{\mathrm{d}x}=0 \quad \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=...$	M1	3.1a
	Equation of tangent at $P(x_1,y_1)$ is $(y-y_1) = -\frac{x_1}{4y_1}(x-x_1)$	M1	3.1a
	$xx_1+4yy_1 = x_1^2+4y_1^2 = 4$ and at $Q(x_2,y_2)$: $xx_2+4yy_2=4$	A1	2.2a
	Intersect at (r,s) gives $rx_1+4sy_1 = 4$ and $rx_2+4sy_2 = 4$	B1	2.1
	Uses their previous results to find the gradient of the line l	M1	3.1a
	$\frac{y_2-y_1}{x_2-x_1} = \frac{-r}{4s}$	A1	1.1b
	Equation of l is $y-y_1=\frac{-r}{4s}(x-x_1)$	M1	2.1
	$4sy+rx = 4sy_1+rx_1 = 4$ *	A1*	2.2a
		(8)	
			(8 marks)

Notes:

M1: Attempts to solve the problem by using differentiation to obtain an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$

M1: Realise the need to form a general equation of the tangent at (x_1,y_1). May use alternative variables

A1: Deduces $x_1^2+4y_1^2=4$ to obtain a correct equation and deduces a correct second equation

B1: Uses (r, s) in both equations to form the two given equations or exact equivalents

M1: Uses their previous results to find the gradient of the line l

A1: $\frac{-r}{4s}$

M1: Formulates the line l with their $\frac{-r}{4s}$. Use of $y-y_1=m(x-x_1)$ or $y=mx+c$ with their gradient and an attempt to find c

A1*: Correct solution leading to $4sy+rx=4sy_1+rx_1$ with deduction that this equals 4 as (x_1,y_1) is on the ellipse. No errors seen

Question	Scheme	Marks	AOs
8(a)	$\mathrm{h}(x)=45+15\sin x+21\sin\left(\frac{x}{2}\right)+25\cos\left(\frac{x}{2}\right)$		
	$\frac{\mathrm{dh}}{\mathrm{d}x}=15\cos x+\frac{21}{2}\cos\left(\frac{x}{2}\right)-\frac{25}{2}\sin\left(\frac{x}{2}\right)$	M1	1.1b
	$\frac{\mathrm{dh}}{\mathrm{d}x}=...+..\frac{1-t^2}{1+t^2}-..\frac{2t}{1+t^2}$	M1	1.1a
	e.g. $\frac{\mathrm{d}h}{\mathrm{d}x}=..\left(2\left(\frac{1-t^2}{1+t^2}\right)^2-1\right)+...$ **or** $\frac{\mathrm{d}h}{\mathrm{d}x}=..\frac{1-\left(\frac{2t}{1-t^2}\right)^2}{1+\left(\frac{2t}{1-t^2}\right)^2}+...$	M1	3.1a
	e.g. $\frac{\mathrm{d}h}{\mathrm{d}x}=15\left(2\left(\frac{1-t^2}{1+t^2}\right)^2-1\right)+\frac{21}{2}\left(\frac{1-t^2}{1+t^2}\right)-\frac{25}{2}\left(\frac{2t}{1+t^2}\right)$	A1	1.1b
	$...=\frac{15[4(1-t^2)^2-2(1+t^2)^2]+21(1-t^2)(1+t^2)-50t(1+t^2)}{2(1+t^2)^2}$ x	M1	2.1
	$...=\frac{9t^4-50t^3-180t^2-50t+51}{2(1+t^2)^2}=\frac{(t^2-6t-17)(9t^2+4t-3)}{2(1+t^2)^2}$ *	A1*	2.1
		(6)	
	8(a) Alternative $\mathrm{h}(x)=...+21\left(\frac{2t}{1+t^2}\right)+25\left(\frac{1-t^2}{1+t^2}\right)$	M1	1.1a
	$=...+15\left[2\left(\frac{2t}{1+t^2}\right)\left(\frac{1-t^2}{1+t^2}\right)\right]+...$ **or** $=...+15\left(\frac{2\left(\frac{2t}{1-t^2}\right)}{1+\left(\frac{2t}{1-t^2}\right)^2}\right)+...$	M1	2.1
	$\mathrm{h}(x)=45+\frac{15(4t(1-t^2))+42t(1+t^2)+25(1-t^4)}{(1+t^2)^2}$	M1	1.1b
	$\mathrm{h}(x)=45-\frac{25t^4+18t^3-102t-25}{(1+t^2)^2}$ or $\frac{20t^4-18t^3+90t^2+102t+70}{(1+t^2)^2}$	A1	1.1b
	$\frac{\mathrm{d}h}{\mathrm{d}x}=\frac{\mathrm{d}h}{\mathrm{d}t}\times\frac{\mathrm{d}t}{\mathrm{d}x}=\frac{('u')(1+t^2)^2-('u')(4t(1+t^2))}{(1+t^2)^4}\times\frac{1}{4}(1+t^2)$	M1	3.1a
	$...=\frac{9t^4-50t^3-180t^2-50t+51}{2(1+t^2)^2}=\frac{(t^2-6t-17)(9t^2+4t-3)}{2(1+t^2)^2}$*	A1*	2.1
		(6)	

Question	Scheme	Marks	AOs
8(b)(i)	Accept any value between $\frac{1}{40} = 0.025$ and $\frac{1}{60} \approx 0.167$ inclusive	B1	3.3
(ii)	Suitable for times since the graphs both oscillate bi-modally with about the same periodicity	B1	3.4
	Not suitable for predicting heights since the heights of the peaks vary over time, but the graph of h(x) has fixed peak height	B1	3.5b
		(3)	
8(c)	Solves at least one of the quadratics $t = \frac{6 \pm \sqrt{36 - 4\times 1\times 17}}{2} = 3 \pm \sqrt{26}$ or $t = \frac{-4 \pm \sqrt{16 - 4\times 9\times(-3)}}{18} = \frac{-2 \pm \sqrt{31}}{9}$	**M1**	1.1b
	Finds corresponding x values, $x = 4\tan^{-1}(t)$ for at least one value of t from the $9t^2 + 4t - 3$ factor	**M1**	1.1b
	One correct value for these x e.g. $x =$ awrt -2.797 or $9.770, 1.510$	**A1**	1.1b
	Maximum peak height occurs at smallest positive value of x, from first graph, but the third of these peaks needed, So $t = 1.509\ldots + 8\pi = 26.642$ is the is the required time	**M1**	3.4
	$x = 26.642$ corresponds to 26 hours and 39 minutes (nearest minute) after 08:00 on 3rd January (Allow if a different greatest peak height used)	**M1**	3.4
	Time of greatest tide height is approximately 10:39 (am) (also allow 10:38 or 10:40)	**A1**	3.2a
		(6)	
			(15 marks)

Notes:

(a)

M1: Differentiates h(x)

M1: Applies t-substitution to both $\left(\frac{x}{2}\right)$ terms with their coefficients

M1: Forms a correct expression in t for the cos x term, using double angle formula and t-substitution, or double 't'-substitution

A1: Fully correct expression in t for $\frac{\mathrm{d}h}{\mathrm{d}x}$

M1: Gets all terms over the correct common factor. Numerators must be appropriate for their terms

A1*: Achieves the correct answer via expression with correct quartic numerator before factorisation

Question 8 notes continued:
Alternative: **(a)** **M1:** Applies t-substitution to both $\left(\frac{x}{2}\right)$ terms **M1:** Forms a correct expression in t for the $\sin x$ term, using double angle formula and t-substitution, or double 't'-substitution **M1:** Gets all terms in t over the correct common factor. Numerators must be appropriate for their terms. May include the constant term too **A1:** Fully correct expression in t for $\mathrm{h}(x)$ **M1:** Differentiates, using both chain rule and quotient rule with their 'u' **A1*:** Achieves the correct answer via expression with correct quartic numerator before factorisation **Note:** The individual terms may be differentiated before putting over a common denominator. In this case score the third M for differentiating with chain rule and quotient rule, then r return to the original scheme
(b)(i) **B1:** Any value between $\frac{1}{40}$ (e.g. taking $\mathrm{h}(0)$ as reference point) **or** $\frac{1}{60}$ (taking lower peaks as reference) **NB:** **Taking high peak as reference gives** $\mathbf{\frac{1}{50}}$
(b)(ii) **B1:** Should mention both the bimodal nature and periodicity for the actual data match the graph of h **B1:** Mentions that the heights of peaks vary in each oscillation
(c) **M1:** Solves (at least) one of the quadratic equations in the numerator **M1:** Must be attempting to solve the quadratic factor from which the solution comes $9t^2 + 4t - 3$ and using $t = \tan\left(\frac{x}{4}\right)$ to find a corresponding value for x **A1:** At least one correct x value from solving the requisite quadratic: awrt any of – 2.797, 1.510, 9.770, 14.076, 22.336, 26.642, 34.902 or 39.208 **M1:** Uses graph of h to pick out their $x = 26.642$ as the time corresponding to the third of the higher peaks, which is the highest of the peaks on 4th January on the tide height graph. As per scheme or allow if all times listed and correct one picked **M1:** Finds the time for one of the values of t corresponding to the highest peaks. E.g. 1.5096...~ 09:31 (3rd January) or 14.076... ~ 22:05 (3rd January) or 26.642…~ 10:39 (4th January) or 39.208…~ 23:13 (4th January). (Only follow through on use of the smallest positive t solution $+ 4k\pi$) **A1:** Time of greatest tide height on 4th January is approximately 10:39. Also allow 10:38 or 10:40